DK-003-001205 Seat No. ________

B. Sc. (Sem. II) (CBCS) Examination

March – 2022

Mathematics : Paper - M - 201

(Geometry, Trigonometry & Matrix Algebra) (Old Course)

Faculty Code : 003
Subject Code : 001205

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.
(2) Right hand side digit indicates the marks of question.
(3) Each section should be written.

SECTION - A

1 Answer all the following short questions : **20**

(1) Write the equation of right circular cylinder with radius r and axis, X-axis.
(2) Define : Cylinder.
(3) Define : Null matrix.
(4) Define : Periodic matrix.
(5) Define : Eigen values of a square matrix A.
(6) If $A=\begin{bmatrix} 1+i & i \\ 2 & 4 \end{bmatrix}$ then $\overline{A}=$ ______.
(7) Define : Bounded above sequence.
(8) Determine the sequence $\{s_n\}=\left\{\frac{1}{n}\right\}$ is convergent or not.
(9) Define : Monotonically increasing sequence.
(10) State Bolzano - weirsstrass theorem.
(11) State Demovre's theorem.
(12) If $x = cis\,\theta$ then $x^n - \frac{1}{x^n} =$ ______.
(13) $\frac{(cis\,\theta)^3\,(cis\,3\theta)^5}{(cis\,5\theta)}$ = ________.

(14) State expansion of sin α in terms of α.

(15) State expansion of $\cos n\theta$ in terms of power of $\cos\theta$ and $\sin\theta$ (where $n \in N$)

(16) Write real part of exp $\left(z^2\right)$.

(17) Define : Hyperbolic sin of x.

(18) $\sec h^2 x + \tan h^2 x =$ _____.

(19) $\log(i) =$ _____.

(20) Write imaginary part of $\cos(x + iy)$.

SECTION - B

2 (a) Attempt any three : **6**

(1) Define : Right circular cylinder, Idempotent matrix.

(2) Check whether matrix $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal.

(3) Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$.

(4) Find eigen values of matrix $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$.

(5) Show that $\lim\limits_{n\to\infty} \frac{1}{n}\left[1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right] = 0$.

(6) Prove that the sequence $\{S_n\}$ is convergent where $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$.

(b) Attempt any three : **9**

(1) Obtain the equation of right circular cylinder with axis $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius r in R^3.

(2) If $AB = A$ and $BA = B$ then prove that A and B are idempotent.

(3) Prove that there exist unique inverse matrix for any non singular matrix.

(4) Prove that eigen value's of Hermitian matrix is real numbers.

(5) Prove that every convergent sequence is bounded.

(6) If $\{a_n\}, \{b_n\}$ be two sequences such that $\lim_{n\to\infty} a_n = a,\ \lim_{n\to\infty} b_n = b$ then prove that $\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$.

(c) Attempt any two : **10**

(1) Obtain equation of cylinder whose generator is parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and enveloping curve is $x^2 + y^2 + z^2 = a^2$.

(2) Verify Cayley - Hamilton theorem for matrix

$$A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}.$$

(3) Solve following linear equations :

$$x + y + z = 9$$
$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$

(4) If $\lim_{n\to\infty} a_n = l$ then show that

$$\lim_{n\to\infty} \left(\frac{a_1 + a_2 + \ldots + a_n}{n} \right) = l.$$

(5) Show that the sequence $\{S_n\}$ defined by $S_1 = \sqrt{2}$ and $S_{n+1} = \sqrt{2S_n}$ converges to 2.

3 (a) Attempt any three : **6**

(1) If $x = \cos\theta + i\sin\theta$ then prove that

$$x^n + \frac{1}{x^n} = 2\cos n\theta.$$

(2) Simplify : $\dfrac{(cis(-5\theta))^3\ (cis\ 3\theta)^4}{(cis\ \theta)^7\ (cis(\theta))^5}$.

(3) If $\sin(6\theta) = a\cos^5\theta\sin\theta + b\cos^2\theta\sin^3\theta +$

$c\cos\theta\cdot\sin^5\theta$ then find a, b and c.

(4) Prove that $\cos h^2 x - \sin h^2 x = 1$.

(5) Separate the real and imaginary part of $\sin h\,(x+1y)$.

(6) Prove that $\log\left(\dfrac{a+ib}{a-ib}\right) = 2i\cdot\tan^{-1}\dfrac{a}{b}$.

(b) Attempt any three : **9**

(1) Evaluate : $(-1)^{1/6}$.

(2) If $\tan\theta = \dfrac{1}{2}$ then find $\tan 6\theta$.

(3) Prove that :

$$\cos^7\theta = \frac{1}{64}(\cos 7\theta + 7\cos 5\theta + 22\cos 3\theta + 35\cos\theta).$$

(4) If $S\cos hz - 13 = 0$ then find value of z.

(5) Prove that $\tan h^{-1} x = \dfrac{1}{2}\log\left(\dfrac{1+x}{1-x}\right)$.

(6) Separate into real and imaginary parts :

$\sin^{-1}(\cos\theta + i\sin\theta)$.

(c) Attempt any two : **10**

(1) State and prove De'Moivere's theorem.

(2) Prove :

$$(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}} = 2\left(a^2+b^2\right)^{\frac{m}{2n}}\cos\left(\frac{m}{n}\tan^{-1}\left(\frac{b}{a}\right)\right)$$

(3) Show that $\cos^3 x = \dfrac{1}{4}\left[4 - \dfrac{x^2}{2!}\left(3^2+3\right) + \dfrac{x^4}{4!}\left(3^4+3\right) + \ldots\right]$

(4) If $\log(x+iy) = 2 - \dfrac{3\pi}{4}i$ then find x and y.

(5) Prove that :

$$\sin h^{-1}(\tan\theta) = \log(\sec\theta + \tan\theta) =$$

$$\cos h^{-1}(\sec\theta) = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right).$$